JHEP HYPER VERSION

Received: *February 28, 2001*, Accepted: *July 20, 2001*
Revised: *June 15, 2001*

Spontaneous CP violation in the next-to-minimal supersymmetric standard model revisited

Gustavo C. Branco,[ab] **Frank Krüger,**[c] **Jorge C. Romão,**[b] **and Ana M. Teixeira**[b]

[a] *Theory Division*
CERN, CH-1211 Geneva 23, Switzerland
[b] *Centro de Física das Interacções Fundamentais (CFIF)*
Departamento de Física, Instituto Superior Técnico
Av. Rovisco Pais, 1049-001 Lisboa, Portugal
[c] *Physik Department, Technische Universität München*
D-85748 Garching, Germany
E-mail: `gbranco@cfif.ist.utl.pt`, `fkrueger@ph.tum.de`, `fromao@alfa.ist.utl.pt`, `ana@cfif.ist.utl.pt`

Abstract: We re-examine spontaneous CP violation at the tree level in the context of the next-to-minimal supersymmetric standard model (NMSSM) with two Higgs doublets and a gauge singlet field. We analyse the most general Higgs potential without a discrete Z_3 symmetry, and derive an upper bound on the mass of the lightest neutral Higgs boson consistent with present experimental data. We investigate, in particular, its dependence on the admixture and CP-violating phase of the gauge singlet field, as well as on $\tan\beta$. To assess the viability of the spontaneous CP violation scenario, we estimate ϵ_K by applying the mass insertion approximation. We find that a non-trivial flavour structure in the soft-breaking A terms is required to account for the observed CP violation in the neutral kaon sector. Furthermore, combining the minimisation conditions for spontaneous CP violation with the constraints coming from K^0–$\bar{K}^0$ mixing, we find that the upper bound on the lightest Higgs-boson mass becomes stronger. We also point out that the electric dipole moments of electron and neutron are a serious challenge for SUSY models with spontaneous CP violation.

Keywords: Spontaneous Symmetry Breaking, Supersymmetric Standard Model, Higgs Physics, CP violation.

Contents

1. Introduction

The origin of CP violation remains a fundamental open question in particle physics. In the standard model (SM), CP is explicitly broken at the lagrangian level through complex Yukawa couplings which lead to CP violation in charged weak interactions via the Cabibbo-Kobayashi-Maskawa (CKM) matrix, $V_{\rm CKM}$ [1]. Although the CKM mechanism can accommodate the experimental value of ϵ_K (and, in principle, also ϵ'/ϵ_K) in the neutral kaon sector, it is not clear whether it is the dominant contribution or the only one. An important motivation to consider new sources of CP violation stems from the fact that within the SM the amount of CP violation may not be sufficient to generate the observed baryon asymmetry in the universe [2].

An alternative scenario for the breaking of CP is to assume that it is a symmetry of the lagrangian which is only spontaneously broken by the vacuum [3]. The purpose of the present paper is to study spontaneous breaking of CP at the tree level within the context of supersymmetry (SUSY). Although the minimal supersymmetric standard model (MSSM) has two Higgs doublets, it is well known that due to SUSY constraints on the Higgs potential it is not possible to obtain spontaneous CP violation (SCPV) at the tree level. The reason for this is entirely analogous to the situation one encounters in non-supersymmetric two-Higgs-doublet models, where SCPV cannot be achieved if a Z_2 symmetry is imposed on the lagrangian in order to guarantee natural flavour conservation in the Higgs sector [4]. The possibility that SCPV in the MSSM might be generated through radiative corrections has been explored in ref. [5], but this particular scenario has already been ruled out by experiment as it inevitably leads to the existence of a very light Higgs boson. Ultimately, this result is a consequence of the Georgi-Pais theorem [6]. It is therefore of interest to consider simple extensions of the MSSM such as a model with at least one gauge singlet field (N) besides the two Higgs doublets ($H_{1,2}$), the so-called next-to-minimal supersymmetric standard model (NMSSM) [7, 8], and to ask if one can achieve spontaneous breaking of CP whilst generating the observed amount of ϵ_K and having Higgs-boson masses that are consistent with experimental data [9]–[13]

Whether or not the observed CP violation in the kaon sector can arise solely from supersymmetry is a serious issue. This possibility has been examined in refs. [14, 15, 16] by simply assuming V_{CKM} to be real, and subsequently investigating whether SUSY sources of CP violation can account for the observed magnitude of ϵ_K and ϵ'/ϵ_K. Here we would like to draw attention to the particularly attractive SUSY scenario with spontaneous CP symmetry breaking. The main point of this class of models is that the reality of the CKM matrix is automatic (see, e.g. refs. [4, 17]), and no longer an *ad hoc* assumption. In the specific scenario we shall be considering, CP violation is caused by the phases ϕ_D and ϕ_N associated with the vacuum expectation values of $\langle H_2^0 \rangle$ and $\langle N \rangle$ respectively. Being a singlet Higgs field, N does not couple to quarks, and although H_2^0 does couple to the up-type quarks, the phase appearing in the quark mass matrix can be rotated away by redefinition of the right-handed quark fields. Consequently, this phase does not appear in the CKM matrix since the W-boson interactions are purely left-handed, but it does enter in other SUSY interactions.

The plan of this paper is as follows. In section 2, we analyse the conditions required for the Higgs potential to have a CP-violating global minimum. We show that the upper bound on the mass of the lightest Higgs boson imposes constraints on the CP-violating phases, ϕ_D and ϕ_N. Section 3 is devoted to a brief review of the real CKM matrix in supersymmetric extensions of the SM with two Higgs doublets and an arbitrary number of gauge singlet fields. In section 4, we introduce the mass and mixing matrices, as well as the couplings of the Higgsino, W-ino, and singlet field in the NMSSM with spontaneous CP violation. The calculation of the relevant SUSY contributions to ϵ_K in the mass insertion approximation is presented in section 5.

We pay particular attention to the case of low $\tan\beta$ and examine the question of whether the choice of parameters of the Higgs potential that is required to obtain spontaneous CP violation is consistent with the experimentally observed indirect CP violation in the kaon sector. The implications of ϵ_K for the upper bound on the lightest Higgs-boson mass and a new flavour structure, besides the one of the CKM matrix, are discussed. We also comment on the electric dipole moments of electron and neutron. Finally, we summarize the key features of our analysis in section 6.

2. The Higgs potential

2.1 The superpotential

We are concerned with the next-to-minimal supersymmetric standard model discussed by Davies *et al.*[18]. The most general form of the superpotential that we analyse here is[1]

$$W = W_{\rm fermion} + W_{\rm Higgs}\,, \tag{2.1}$$

with

$$W_{\rm fermion} = \varepsilon_{ab}\left(h_U^{ij}\widehat{Q}_i^a\widehat{U}_j\widehat{H}_2^b + h_D^{ij}\widehat{Q}_i^b\widehat{D}_j\widehat{H}_1^a + h_E^{ij}\widehat{L}_i^b\widehat{R}_j\widehat{H}_1^a\right), \tag{2.2}$$

$$W_{\rm Higgs} = -\lambda\widehat{N}\varepsilon_{ab}\widehat{H}_1^a\widehat{H}_2^b - \frac{k}{3}\widehat{N}^3 - r\widehat{N} - \mu\varepsilon_{ab}\widehat{H}_1^a\widehat{H}_2^b, \tag{2.3}$$

where $i,j = 1,2,3$ denote generation indices, $a,b = 1,2$ are SU(2) indices, and ε is a completely antisymmetric 2×2 matrix with $\varepsilon_{12} = 1$. In the above expression, $\widehat{H}_1^a$ and $\widehat{H}_2^a$ denote the Higgs doublets of the minimal supersymmetric standard model and $\widehat{N}$ is a singlet field. The matrices h_U, h_D, and h_E give rise to the usual Yukawa interactions which generate the masses of quarks and leptons. Since we are dealing here with spontaneous breaking of CP rather than explicit CP violation in the Higgs sector, these matrices are real.

2.2 Soft-breaking terms

In addition to the superpotential given by eq. (2.1), we have to specify the SUSY soft-breaking terms. These are given by

$$\mathcal{L}_{\rm SB} = \mathcal{L}_{\rm SB}^{\rm fermion} + \mathcal{L}_{\rm SB}^{\rm Higgs}\,, \tag{2.4}$$

where

$$\begin{aligned}-\mathcal{L}_{\rm SB}^{\rm fermion} = {} & M_{Q_{ij}}^2\widetilde{Q}_i^{a*}\widetilde{Q}_j^a + M_{U_{ij}}^2\widetilde{U}_i\widetilde{U}_j^* + M_{D_{ij}}^2\widetilde{D}_i\widetilde{D}_j^* + M_{L_{ij}}^2\widetilde{L}_i^{a*}\widetilde{L}_j^a + \\ & + M_{R_{ij}}^2\widetilde{R}_i\widetilde{R}_j^* - \frac{1}{2}\left(M_s\lambda_s\lambda_s + M\lambda\lambda + M'\lambda'\lambda' + {\rm H.c.}\right) + \\ & + \varepsilon_{ab}\left(A_U^{ij}h_U^{ij}\widetilde{Q}_i^a\widetilde{U}_jH_2^b + A_D^{ij}h_D^{ij}\widetilde{Q}_i^b\widetilde{D}_jH_1^a + A_E^{ij}h_E^{ij}\widetilde{L}_i^b\widetilde{R}_jH_1^a\right),\end{aligned} \tag{2.5}$$

[1] This form of the superpotential has also recently been studied in ref. [19], within the context of electroweak baryogenesis.

and

$$-\mathcal{L}_{\rm SB}^{\rm Higgs} = m_{H_1}^2 H_1^{a*} H_1^a + m_{H_2}^2 H_2^{a*} H_2^a + m_N^2 N^* N - \\ - \left(B\mu \varepsilon_{ab} H_1^a H_2^b + A_\lambda N \varepsilon_{ab} H_1^a H_2^b + \frac{A_k}{3} N^3 + A_r N + \text{H.c.} \right) . \quad (2.6)$$

2.3 The scalar potential and spontaneous CP violation

Following ref. [18], we do not require the superpotential to be invariant under a discrete Z_3 symmetry which would imply $\mu = r = 0$ (thereby solving the so-called 'μ problem' of the MSSM).[2] Further, we do not relate the soft SUSY-breaking parameters to some common unification scale but rather take them as arbitrary at the electroweak scale.

As it was noted by one of the authors (J.C.R.) a long time ago [12], the NMSSM with a Z_3 symmetry has no spontaneous CP violation (no-go theorem). The inclusion of the Z_3-breaking terms in eq. (2.1), on the other hand, evades that no-go theorem, as was shown by Pomarol [13]. Throughout we shall assume that the tree-level potential is CP conserving and take all parameters real, but allow complex vacuum expectation values (VEV's) for the neutral Higgs fields which emerge after spontaneous symmetry breaking:

$$\langle H_1^0 \rangle = \frac{v_1}{\sqrt{2}} e^{i\theta_1} , \quad \langle H_2^0 \rangle = \frac{v_2}{\sqrt{2}} e^{i\theta_2} , \quad \langle N \rangle = \frac{v_3}{\sqrt{2}} e^{i\theta_3} . \quad (2.7)$$

From the superpotential and soft supersymmetry breaking terms, eqs. (2.3) and (2.6), we derive the following CP-invariant neutral scalar potential:

$$V = V_0 + \frac{1}{8}(\lambda_1 v_1^4 + \lambda_2 v_2^4) + \frac{1}{4}[(\lambda_3 + \lambda_4) v_1^2 v_2^2 + (\lambda_5 v_1^2 + \lambda_6 v_2^2) v_3^2] + \\ + \frac{1}{2}\lambda_7 v_1 v_2 v_3^2 \cos(\theta_1 + \theta_2 - 2\theta_3) + \frac{1}{4}\lambda_8 v_3^4 + \frac{1}{\sqrt{2}}\lambda\mu(v_1^2 + v_2^2) v_3 \cos(\theta_3) + \\ + \frac{1}{2}(m_1^2 v_1^2 + m_2^2 v_2^2 + m_3^2 v_3^2) - \frac{1}{\sqrt{2}} m_4 v_1 v_2 v_3 \cos(\theta_1 + \theta_2 + \theta_3) - \\ - \frac{1}{3\sqrt{2}} m_5 v_3^3 \cos(3\theta_3) + m_6^2 v_1 v_2 \cos(\theta_1 + \theta_2) + m_7^2 v_3^2 \cos(2\theta_3) + \\ + \sqrt{2} m_8^3 v_3 \cos(\theta_3) , \quad (2.8)$$

where $V_0 = r^2$ and

$$\lambda_1 = \lambda_2 = \frac{1}{4}(g^2 + g'^2) , \quad \lambda_3 = -\frac{1}{4}(g^2 + g'^2 - 4\lambda^2) , \quad \lambda_4 = 0 , \\ \lambda_5 = \lambda_6 = \lambda^2 , \quad \lambda_7 = \lambda k , \quad \lambda_8 = k^2 , \quad (2.9)$$

$$m_{1,2}^2 = m_{H_{1,2}}^2 + \mu^2 , \quad m_3^2 = m_N^2 , \quad m_4 = A_\lambda , \quad m_5 = A_k , \\ m_6^2 = -B\mu + \lambda r , \quad m_7^2 = kr , \quad m_8^3 = -A_r . \quad (2.10)$$

[2] We remind the reader that a spontaneously broken discrete symmetry may lead to cosmological domain-wall problems [20]. We do not pursue this subject here.

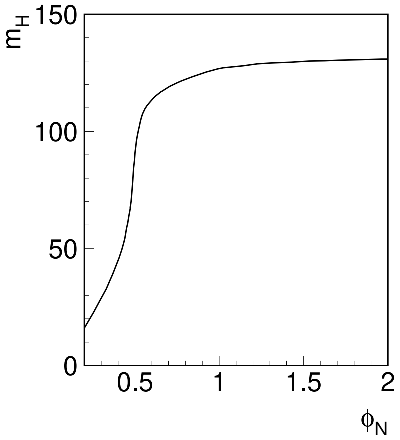

Figure 1: Maximum value of the lightest Higgs-boson mass (in GeV) as a function of the CP-violating phase ϕ_N (in radians) of the Higgs singlet field.

In the numerical analysis reported here, we take the λ_l's ($l = 1, 2, \ldots, 8$) as given by eq. (2.9), i.e. in terms of g, g', λ, k, but consider the m_l's as arbitrary parameters. Referring to eq. (2.8), we see that the doublet phases enter only in the combination $\theta_1 + \theta_2$, and thus we may define

$$\phi_D = \theta_1 + \theta_2, \quad \phi_N = \theta_3 \tag{2.11}$$

for the doublet and singlet phase respectively. We shall henceforth set $\theta_1 = 0$ in eq. (2.11), without loss of generality.

2.4 Mass spectrum

To obtain the physical masses of the NMSSM with SCPV, the following procedure has been adopted. We randomly choose sets of parameters and perform the minimisation numerically. Next, we check that the local minimum is a true one not only by verifying the minimisation conditions, but also by ensuring that all the physical Higgs bosons have positive squared masses. In calculating the neutral Higgs-boson mass matrix, radiative corrections due to top-stop loops have been included. Finally, we require that the chargino and squark masses are above the current experimental lower limits [21].

We find that an acceptable mass spectrum can be easily obtained, with the exact values depending on the set of parameters we choose. Rather than presenting tables or plots for different sets of parameters, we will concentrate on two issues: (a) the influence of the CP-violating phases on the lightest Higgs-boson mass; and (b) the maximum theoretically attainable value of its mass. As far as the former is concerned, we confirm the results of an analysis of the Higgs potential performed in ref. [18], where it was pointed out that the large-phase solution is favoured. This feature can be clearly observed in figure 1, where we display the Higgs-boson mass, m_{H^0}, as a function of the phase of the gauge singlet field.

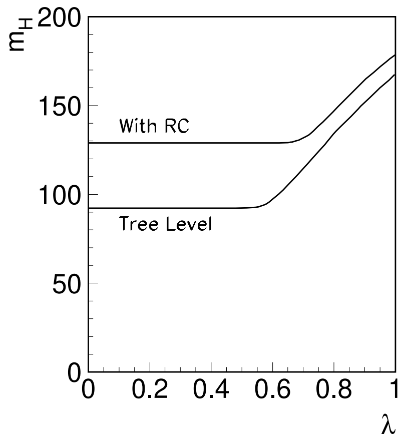

Figure 2: Maximum value of the lightest Higgs-boson mass (in GeV) vs λ at the tree level and after including radiative corrections (at one-loop level) for $M_{\rm SUSY} = 1\,\text{TeV}$.

It is important to note that large CP phases may be in conflict with existing limits on the electric dipole moments (EDM's) of electron and neutron. However, these constraints can be evaded, for instance, if substantial cancellations among different SUSY contributions do occur [22, 23] or if non-universal soft-breaking terms are present [24]. (We will return to this point in a subsequent section.)

As for the maximal possible value of the Higgs-boson mass, the result can differ from that of the MSSM for the case of large values of the coupling constant λ. In figure 2, we show the tree level and one-loop corrected Higgs-boson mass as a function of λ.

It is apparent that only for large values of the coupling λ the situation is different from that of the MSSM. For low values of λ, corrections to the tree level Higgs-boson mass are significant and depend mainly on the SUSY scale that we take for the squarks. This is illustrated in figure 3 where we plot the upper limit on m_{H^0} versus $\tan\beta \equiv v_2/v_1$ [cf. eq. (2.7)] for different values of M_{SUSY}.

Before leaving the subject of the Higgs potential, it should be emphasized that the Higgs boson mass limits obtained at LEP for the standard model and the MSSM (see, e.g. ref. [25]) do not necessarily apply to the NMSSM because the lightest neutral Higgs boson may have a reduced coupling to the Z^0 due to some singlet admixture [7, 26]. In this case, the Higgs boson might not be detectable.

3. A real CKM matrix

We outline briefly the scenario of a real CKM matrix in the framework of supersymmetry with SCPV. In this case, CP invariance is imposed on the lagrangian, and consequently all couplings are real. To address the question of whether the CKM mechanism occurs in the NMSSM model, let us consider the quark Yukawa couplings.

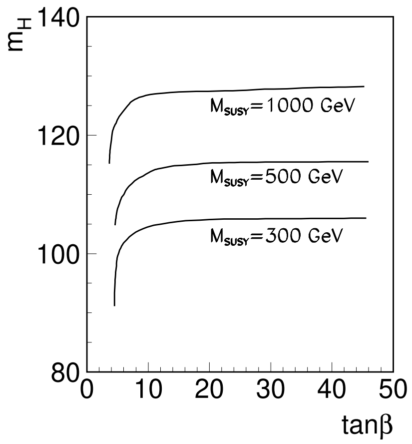

Figure 3: Upper bound on the lightest Higgs-boson mass (in GeV) as a function of $\tan\beta$ for various values of $M_{\rm SUSY}$.

As mentioned in the introduction, the gauge singlet field does not couple to quarks. Thus, the lagrangian in terms of the weak eigenstates may be written as (omitting generation indices)

$$\mathcal{L}_Y = -h_U H_2^{0*} \bar{u}_L u_R - h_D H_1^{0*} \bar{d}_L d_R + \text{H.c.}\,, \tag{3.1}$$

where $h_{U,D}$ are arbitrary real matrices in flavour space. After spontaneous symmetry breaking, the up- and down-type quarks acquire masses, namely

$$m_U = h_U \frac{v_2}{\sqrt{2}} e^{-i\phi_D}\,, \quad m_D = h_D \frac{v_1}{\sqrt{2}}\,. \tag{3.2}$$

Note that the overall phase ϕ_D can be rotated away by means of a phase transformation on u_R, i.e. $u_R \to u'_R = e^{-i\phi_D} u_R$. Since the W-boson interactions are purely left-handed, this phase does not show up in the charged weak interactions. Consequently, the CKM matrix is real and CP violation arises solely from the relative phases that appear in the VEV's of the neutral Higgs fields, eq. (2.7). An alternative way to derive the above result is to compute the weak basis invariant $T \equiv \text{Tr}[H_U, H_D]^3$, where $H_U \equiv m_U m_U^\dagger$ and $H_D \equiv m_D m_D^\dagger$ [27]. It follows from eq. (3.2) that T vanishes, and hence there is no CP violation through the CKM mechanism in our model.

4. Content of the NMSSM

As we have argued in the previous section, all supersymmetry CP-violating phases are equal to zero and the CKM matrix is real. Even so, the phases associated with the VEV's, ϕ_D and ϕ_N, appear in the scalar quark, gaugino and Higgsino mass matrices, as well as in some of the vertices. We first consider the squark mass matrices.

4.1 Squark mass matrices

The 6×6 squark mass-squared matrices in the $(\tilde{q}_L, \tilde{q}_R)$ basis are given by

$$M_{\tilde{q}}^2 = \begin{pmatrix} M_{\tilde{q}_{LL}}^2 & M_{\tilde{q}_{LR}}^2 \\ M_{\tilde{q}_{RL}}^2 & M_{\tilde{q}_{RR}}^2 \end{pmatrix}, \qquad \tilde{q} = \tilde{U}, \tilde{D}, \tag{4.1}$$

made up of 3×3 submatrices. It proves convenient to work in the so-called 'super-CKM' basis, in which the quark mass matrices are diagonal, and squarks as well as quarks are rotated simultaneously (see ref. [28] for details). In this basis, the aforementioned submatrices are of the form

$$\begin{aligned}
M_{\tilde{U}_{LL}}^2 &= (m_U^{\rm diag})^2 + V_L^U M_{\tilde{Q}}^2 V_L^{U\dagger} + \frac{1}{6} M_Z^2 \cos 2\beta (3 - 4\sin^2\theta_W) \mathbb{1}\,, \\
M_{\tilde{U}_{RR}}^2 &= (m_U^{\rm diag})^2 + V_R^U M_{\tilde{U}_R}^2 V_R^{U\dagger} + \frac{2}{3} M_Z^2 \cos 2\beta \sin^2\theta_W \mathbb{1}\,, \\
M_{\tilde{U}_{LR}}^2 &= M_{\tilde{U}_{RL}}^{2\dagger} = V_L^U Y_U^* V_R^{U\dagger} \frac{v_2}{\sqrt{2}} - \mu_{\rm eff} \cot\beta e^{i\phi_D} m_U^{\rm diag}\,, \\
M_{\tilde{D}_{LL}}^2 &= (m_D^{\rm diag})^2 + V_L^D M_{\tilde{Q}}^2 V_L^{D\dagger} - \frac{1}{6} M_Z^2 \cos 2\beta (3 - 2\sin^2\theta_W) \mathbb{1}\,, \\
M_{\tilde{D}_{RR}}^2 &= (m_D^{\rm diag})^2 + V_R^D M_{\tilde{D}_R}^2 V_R^{D\dagger} - \frac{1}{3} M_Z^2 \cos 2\beta \sin^2\theta_W \mathbb{1}\,, \\
M_{\tilde{D}_{LR}}^2 &= M_{\tilde{D}_{RL}}^{2\dagger} = V_L^D Y_D^* V_R^{D\dagger} \frac{v_1}{\sqrt{2}} - \mu_{\rm eff} \tan\beta e^{i\phi_D} m_D^{\rm diag}\,,
\end{aligned} \tag{4.2}$$

θ_W being the Weinberg angle, $\mathbb{1}$ denotes a 3×3 unit matrix, and the V's diagonalize the up- and down-type quark mass matrices in eq. (3.2). [Note that the V matrices are orthogonal, and therefore $V_{\rm CKM} = V_L^U (V_L^D)^{\rm T}$.] In what follows, we take a particular case of the 'super-CKM' basis where $V_L^D = \mathbb{1}$ and $V_L^U = V_{\rm CKM}$, so that

$$(m_U^{\rm diag})^2 = V_{\rm CKM} (m_U m_U^\dagger) V_{\rm CKM}^{\rm T}\,. \tag{4.3}$$

Further, we have introduced the shorthand notation

$$Y_q^{ij} \equiv A_q^{ij} h_q^{ij}\,, \qquad q = U, D \qquad (\text{no sum over } i, j)\,, \tag{4.4}$$

$$\mu_{\rm eff} \equiv \mu + \lambda \frac{v_3}{\sqrt{2}} e^{i\phi_N}\,. \tag{4.5}$$

[Notice that eqs. (4.2)–(4.5) show explicitly the dependence on the CP phases.] As we shall see in the next section, a non-universal flavour structure in the A terms, i.e. $A_q^{ij} \neq$ constant, is indispensable for having sizable supersymmetry contributions to CP violation in the kaon sector.[3] At this stage, we therefore make no assumptions regarding the nature of the several matrices and couplings involved.

The squark mass-squared matrix, $M_{\tilde{q}}^2$, can then be diagonalized by a 6×6 unitary matrix such that

$$(M_{\tilde{q}}^{\rm diag})^2 = R_{\tilde{q}} M_{\tilde{q}}^2 R_{\tilde{q}}^\dagger\,. \tag{4.6}$$

[3] For related work, see refs. [14, 15].

4.2 Chargino mass matrix

We now turn our attention to the four-component Dirac fermions, $\tilde{\chi}^{\pm}_{1,2}$, which arise from the mixing of the W-inos and the charged Higgsinos. Within SUSY, the chargino mass terms in the lagrangian are

$$\mathcal{L}_{\rm mass} = -\frac{1}{2}(\psi^{+T}, \psi^{-T}) \begin{pmatrix} 0 & M_{\tilde{\chi}}^T \\ M_{\tilde{\chi}} & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{H.c.}\,, \tag{4.7}$$

where $\psi^{+T} = (-i\lambda^+, \tilde{H}_2^+)$, $\psi^{-T} = (-i\lambda^-, \tilde{H}_1^-)$. The chargino mass matrix in the NMSSM with spontaneous CP violation reads

$$M_{\tilde{\chi}} = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta e^{-i\phi_D} \\ \sqrt{2} M_W \cos\beta & \mu_{\rm eff} \end{pmatrix} \tag{4.8}$$

with the gaugino mass M_2 being real and $\mu_{\rm eff}$ given in eq. (4.5). As usual, $M_{\tilde{\chi}}$ can be diagonalized by means of a biunitary transformation, i.e.

$$M_{\tilde{\chi}}^{\rm diag} = U^* M_{\tilde{\chi}} V^\dagger\,, \tag{4.9}$$

where $M_{\tilde{\chi}}^{\rm diag}$ is positive and diagonal, and U, V are unitary matrices.

For convenience, we perform our calculation in the weak (rather than the physical) basis, i.e. $\tilde{W}^- = (-i\lambda^-, i\overline{\lambda}^+)^{\rm T}$, $\tilde{H}^- = (\tilde{H}_1^-, \overline{\tilde{H}}_2^+)^{\rm T}$, and their relation can be summarized as follows:[4]

$$\begin{aligned} P_L \tilde{W}^- &= P_L(U_{11}^* \tilde{\chi}_1^- + U_{21}^* \tilde{\chi}_2^-), \quad & P_R \tilde{W}^- &= P_R(V_{11}\tilde{\chi}_1^- + V_{21}\tilde{\chi}_2^-), \qquad (4.10\text{a}) \\ P_L \tilde{H}^- &= P_L(U_{12}^* \tilde{\chi}_1^- + U_{22}^* \tilde{\chi}_2^-), \quad & P_R \tilde{H}^- &= P_R(V_{12}\tilde{\chi}_1^- + V_{22}\tilde{\chi}_2^-), \qquad (4.10\text{b}) \end{aligned}$$

with $P_{L,R} = (1 \mp \gamma_5)/2$. In order to specify the relevant couplings within the NMSSM, we recall that [29]

$$-\mathcal{L}^{\tilde{W},\tilde{H}} = g(H_1^{0*}\overline{\tilde{W}} P_L \tilde{H} + H_2^{0*}\overline{\tilde{H}} P_L \tilde{W}) + \lambda N \overline{\tilde{H}} P_L \tilde{H} + \text{H.c.} \tag{4.11}$$

Then, substituting the VEV's from eq. (2.7), and adding W-ino and Higgsino 'mass' terms, we arrive at

$$-\mathcal{L}_{\rm int} = m_{\tilde{W}} \overline{\tilde{W}}\tilde{W} + m_{\tilde{H}}\overline{\tilde{H}}\tilde{H} + \frac{g}{\sqrt{2}}(v_1 e^{-i\varphi}\overline{\tilde{W}}_R \tilde{H}_L + v_2 e^{i\phi_D}\overline{\tilde{W}}_L \tilde{H}_R + \text{H.c.})\,, \tag{4.12}$$

where

$$m_{\tilde{W}} = M_2\,, \qquad m_{\tilde{H}} = |\mu_{\rm eff}|\,, \qquad \varphi = \arg(\mu_{\rm eff})\,, \tag{4.13}$$

and $\mu_{\rm eff}$ as in eq. (4.5). In deriving eq. (4.12), we have adjusted our definition of the phase of the left-handed Higgsino field such that $\tilde{H}_L \to e^{-i\varphi}\tilde{H}_L$. As for gluino and neutralino interactions, we will argue in the next section that to good approximation the chargino box diagram may be regarded as the dominant contribution to ϵ_K (see also refs. [11, 13]).

[4]We define $\tilde{W}^-$ and $\tilde{H}^-$ as particles, contrary to ref. [29].

5. Implications of indirect CP violation for the NMSSM

In this section, we wish to explore the consequences of SCPV for the upper bound of the lightest Higgs-boson mass derived in section 2 by taking into account CP violation in K^0–$\bar{K}^0$ mixing. To accomplish this, we will compute the box-diagram contributions to ϵ_K by applying the mass insertion approximation [30]. That is, adopting the notation

$$M_{\tilde{q}}^2 \equiv (M_{\tilde{q}}^0)^2 + M_{\tilde{q}}'^2, \quad (M_{\tilde{q}}^0)^2 = \mathrm{diag}(a_1^0, \ldots, a_6^0), \tag{5.1}$$

and recalling eq. (4.6), we may write to first order in $M_{\tilde{q}}'^2$ [31]

$$(R_{\tilde{q}}^\dagger)_{mk} f(a_k)(R_{\tilde{q}})_{kn} = \delta_{mn} f(a_n^0) + (M_{\tilde{q}}'^2)_{mn} F(a_m^0, a_n^0)\,, \tag{5.2}$$

f being an arbitrary function, $k = 1, \ldots, 6$, and

$$F(x,y) = \frac{f(x) - f(y)}{x - y}\,. \tag{5.3}$$

5.1 Effective hamiltonian

Let us start with the effective hamiltonian governing $\Delta S = 2$ transitions, which can be written in the form [32]:

$$H_{\mathrm{eff}} = \sum_i c_i \mathcal{O}_i\,, \tag{5.4}$$

where the c_i's are the Wilson coefficients describing the short-distance interactions while $\mathcal{O}_i$ denote local operators which may be found, e.g. in refs. [33, 34]. The off-diagonal element of the neutral kaon mass matrix is related to H_{eff} through

$$\mathcal{M}_{12} = \frac{\langle K^0 | H_{\mathrm{eff}} | \bar{K}^0 \rangle}{2 m_K}\,, \tag{5.5}$$

and its imaginary part gives rise to (assuming $\epsilon' \ll \epsilon_K$)

$$\epsilon_K \simeq \frac{e^{i\pi/4}}{\sqrt{2}} \frac{\Im \mathcal{M}_{12}}{\Delta m_K}\,, \tag{5.6}$$

with the experimental values of $\Delta m_K = (3.489 \pm 0.008) \times 10^{-12}$ MeV[5] and $|\epsilon_K| = (2.271 \pm 0.017) \times 10^{-3}$ [21].

The Wilson coefficients c_i in the presence of SUSY contributions can be decomposed as follows:

$$c_i = c_i^W + c_i^{H^\pm} + c_i^{\tilde{\chi}^\pm} + c_i^{\tilde{g}} + c_i^{\tilde{\chi}^0}\,. \tag{5.7}$$

[5]Note that for Δm_K, and following standard procedure, we have used the experimental value. No useful bound on the present model can be derived from $\Delta m_K|_{\mathrm{exp}}$, since it receives important long-distance contributions.

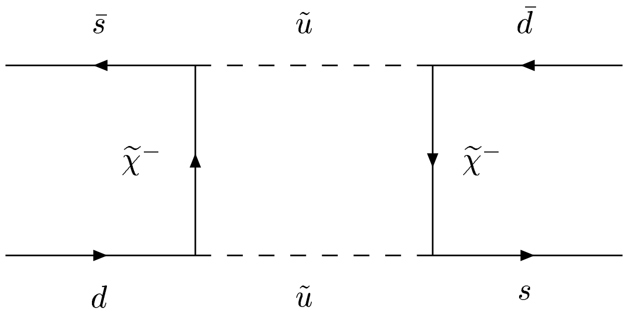

Figure 4: The dominant box-diagram contribution to the off-diagonal element $\Im\mathcal{M}_{12}$ in the neutral kaon mass matrix within the framework of the NMSSM.

Regarding the various contributions in eq. (5.7), we confine ourselves to a relatively few remarks. First, there are no W-boson and charged Higgs-boson contributions to the imaginary part of $\mathcal{M}_{12}$ since the CKM matrix is real. Second, in subsequent calculations we will choose a basis where the down-type Yukawa matrices are diagonal. As a consequence, the flavour off-diagonal gluino contributions are either zero or real in the approximation of retaining only a single mass insertion in an internal squark line. Third, box diagrams with neutralinos are expected to compete with the chargino contributions only for large values of $\tan\beta$ (i.e. $\tan\beta \sim 50$), and hence are less important for the present scenario with low $\tan\beta$. In the remainder of this section, we will therefore focus on the chargino box diagram with scalar up-type quarks (see figure 4) as it gives by far the dominant contribution in our model.

Turning to the operators $\mathcal{O}_i$ in eq. (5.4), the $\Delta S = 2$ transition within the model under study is largely governed by the V–A four-fermion operator $\mathcal{O}_1 = \overline{d}\gamma^\mu P_L s \overline{d}\gamma_\mu P_L s$. As a matter of fact, the new-physics contribution to the remaining operators (i.e. scalar and pseudoscalar operators) are suppressed relative to the chargino contribution either through small quark masses, $m_{s,d}$, or small Yukawa couplings. Thus, it is fairly reasonable to consider only the non-standard contribution to the Wilson coefficient c_1. The relevant hadronic matrix element in the vacuum saturation approximation is given by $\langle K^0|\mathcal{O}_1|\bar{K}^0\rangle = (2/3) f_K^2 m_K^2$, with $f_K = 160$ MeV.

5.2 Chargino box diagram

In the $\tilde{W}$-$\tilde{H}$ basis, and making use of the mass insertion approximation, the chargino contributions to the short-distance coefficient c_1 can be written as a sum of 16 individual terms, each corresponding to a particular box diagram. Introducing the notation $\{ij\}$ $(i, j = L, R)$ for the various mass insertions, these contributions can be classified in terms of the nature of the mass insertion in the internal squark line and the number of $\tilde{W}$-$\tilde{H}$ exchanges in the sfermion line. That is, the relevant amplitude

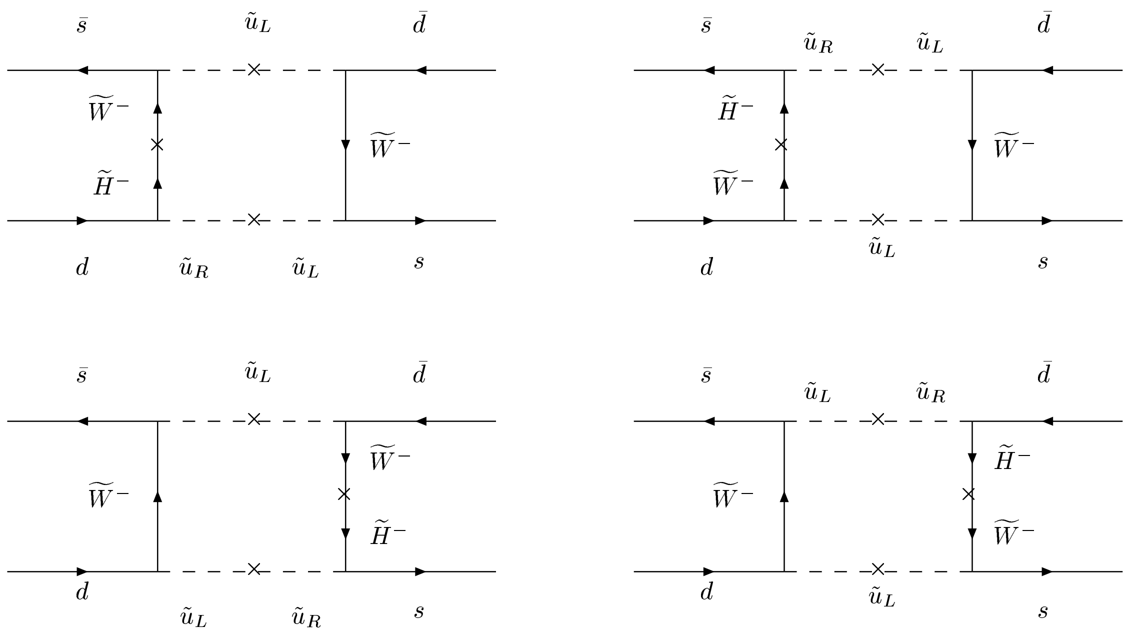

Figure 5: The main contributions to ϵ_K in the mass insertion approximation with W-ino and Higgsino exchange.

for c_1 can be symbolically written as

$$A_{c_1}^{\tilde{\chi}^\pm} \sim (X_{\tilde{W}-\tilde{H}})^0[\{LL\}\{LL\} + \{RR\}\{RR\} + \{LR\}\{RL\}] + \\ +(X_{\tilde{W}-\tilde{H}})^1[\{LL\}\{LR\} + \{LR\}\{RR\}] + \\ +(X_{\tilde{W}-\tilde{H}})^2[\{LL\}\{RR\} + \{LR\}\{LR\}] \,. \tag{5.8}$$

Only one diagram belongs to the $\{LL\}\{LL\}$ and $\{RR\}\{RR\}$ mass insertions, whereas we find two (four) diagrams associated with the $\{LR\}\{RL\}$, $\{LL\}\{RR\}$, $\{LR\}\{LR\}$ ($\{LL\}\{LR\}$, $\{LR\}\{RR\}$) mass insertions. It turns out that the dominant contribution to c_1 corresponds to the class of diagrams with $(X_{\tilde{W}-\tilde{H}})^1\{LL\} \times \{LR\}$, depicted in figure 5, in agreement with the findings of refs. [11, 13].

The calculation of the dominant contributions to $\Im\mathcal{M}_{12}$ for the case of non-degenerate left-handed up-squarks leads to a cumbersome expression. Hence, for the sake of simplicity, we perform the calculation in the limit of degenerate left-handed up-type squarks, and find that the expression for $\Im\mathcal{M}_{12}$ becomes much simpler, and its physical interpretation more transparent. (We have numerically verified that small values of the off-diagonal elements of $M^2_{\tilde{Q}}$ have little effect on the $\tilde{u}_L$ mass splitting.) In that limit, i.e. $m_{\tilde{u}_L} = \langle m_{\tilde{q}} \rangle$, where $\langle m_{\tilde{q}} \rangle$ represents the average squark mass of the first two generations, we obtain

$$\Im\mathcal{M}_{12} = \frac{2G_F^2 f_K^2 m_K m_W^4}{3\pi^2 \langle m_{\tilde{q}} \rangle^8}(V_{td}^* V_{ts}) m_t^2 \left| e^{i\phi_D} m_{\tilde{W}} + \cot\beta\, m_{\tilde{H}} \right| \times \\ \times \left\{ \Delta A_U \sin(\varphi_\chi - \phi_D)\, \left(M^2_{\tilde{Q}}\right)_{12} I(r_{\tilde{W}}, r_{\tilde{H}}, r_{\tilde{u}_L}, r_{\tilde{t}_R}) \right\} , \tag{5.9}$$

with $m_{\tilde{H},\tilde{W}}$ and $\left(M^2_{\tilde{Q}}\right)_{12}$ as defined in eqs. (4.13) and (4.2) respectively. Here we have used unitarity of the CKM matrix and kept only the leading top-quark contribution. In the above formula, $r_i = m_i^2/\left\langle m_{\tilde{q}}\right\rangle^2$, $\Delta A_U \equiv A_U^{13} - A_U^{23}$, and

$$\varphi_\chi = \arg\left(e^{i\phi_D} m_{\tilde{W}} + \cot\beta m_{\tilde{H}}\right). \tag{5.10}$$

The function I in eq. (5.9) can be reduced successively to elementary functions for appropriate limits of its arguments (see refs. [23, 31]). For ease of writing, we give the result for the special case

$$I(x,x,1,1) = \frac{1}{2}\left[\frac{10+19x+x^2}{3(1-x)^5} + \frac{(1+6x+3x^2)}{(1-x)^6}\ln x\right], \tag{5.11}$$

which corresponds to the scenario with $m_{\tilde{H}} = m_{\tilde{W}}$ and degenerate squarks. The appearance of A_U^{i3} $(i = 1, 2)$ terms in eq. (5.9) is closely related to having the Higgsino coupling to down and strange quarks in distinct diagrams, and stems from the trilinear soft-breaking terms in $Y_U^{ij} \equiv A_U^{ij} h_U^{ij}$ [eq. (4.4)]. From inspection of eq. (5.9), it is straightforward to conclude that in order to get $\Im\mathcal{M}_{12}$ different from zero we need to move aside from a theory of universal A_U terms (i.e. $\Delta A_U \neq 0$) – in other words, it is not possible to saturate the observed CP violation in the K-meson system in the context of SUSY with a real CKM matrix and universal A_U terms [14, 15].[6] Let us note parenthetically that the need for a special A_U^{i3} $(i = 1, 2)$ texture as a key element to get the experimentally measured ϵ_K has also been pointed out in ref. [14], although in a different scenario.

Since the sign of ΔA_U is not fixed, we do not include the constraint $\Re\epsilon_K > 0$ in the analysis which follows.

5.3 Numerical results and discussion

For our numerical calculations, we have used the nominal values

$$\left(M^2_{\tilde{Q}}\right)_{12}/\left\langle m_{\tilde{q}}\right\rangle^2 = 0.08, \quad V_{td} = 0.0066, \quad V_{ts} = -0.04, \quad m_t = 175\,\mathrm{GeV}. \tag{5.12}$$

Notice that the value of V_{td} differs from the one determined in the context of the SM with CP violation by fitting to experimental data; this is due to the fact that we are dealing with a flat triangle and the corresponding orthogonality constraints.[7] The remaining parameters are evaluated numerically by minimising the Higgs potential (see section 2). Our results for the absolute value of ϵ_K for various sets of SUSY

[6] It has recently been pointed out in ref. [35] that there might be a connection between a non-trivial flavour structure and sizable SUSY CP violation, thereby avoiding domain-wall problems.

[7] The value used for $|V_{td}|$ is consistent with the experimental value of B_d^0–$\bar{B}_d^0$ mixing, provided one assumes a new-physics contribution to Δm_{B_d} of at least 20%. To be specific, we have chosen a positive value for ρ. As for a negative value of ρ, orthogonality of the CKM matrix would imply $|V_{td}| = 0.011$.

$\|\epsilon_K\|$ (10^{-3})	ϕ_D (rad)	ϕ_N (rad)	m_{H^0} (GeV)	$\langle m_{\tilde q}\rangle$ (GeV)	$m_{\tilde t_R}$ (GeV)	$\tan\beta$	λ	v_3 (GeV)
3.24	4.71	1.57	99	252	235	6.7	−0.03	327
3.03	0.89	1.75	97	261	168	6.6	+0.33	387
2.75	4.71	4.71	99	232	201	9.2	−0.02	221
2.42	1.96	4.08	94	299	174	5.1	−0.06	352
2.10	4.67	4.75	98	279	220	7.8	+0.01	142
2.02	4.68	4.71	92	250	152	7.4	+0.02	371
2.01	4.18	4.73	96	280	232	4.6	−0.01	238
1.31	1.12	4.72	100	273	241	9.6	−0.01	238
1.29	2.35	4.70	99	258	230	6.1	−0.13	363

Table 1: Numerical values of $|\epsilon_K|$ in the low $\tan\beta$ region for certain sets of model parameters that satisfy the minimisation condition of the Higgs potential. We have chosen $\Delta A_U = 500\,\text{GeV}$ for the non-universal A terms, as described in the text.

parameters and low $\tan\beta$ are reported in Table 1. Recall that the numerical values we present for $|\epsilon_K|$ have been derived for degenerate left-handed squarks. Taking into account corrections due to non-degeneracy will lead to a very small enhancement, typically a few percent, for $\Im\mathcal{M}_{12}$. Hence, the values for $|\epsilon_K|$ listed in table 1 may be interpreted as a lower bound.

As far as M_2 and μ are concerned, we obtain the ranges $90\,\text{GeV} \lesssim M_2 \lesssim 160\,\text{GeV}$ and $120\,\text{GeV} \lesssim |\mu| \lesssim 270\,\text{GeV}$. Turning to the non-universal flavour structure, it is obvious from eq. (5.9) that there is a linear dependence of ϵ_K on the relative difference ΔA_U. In order to saturate the value of $|\epsilon_K|$ and to obey present experimental limits on the sparticle spectrum, one has to take ΔA_U of order 500 GeV. A detailed discussion of scenarios where such values for A_U^{i3} $(i = 1, 2)$ appear lies beyond the scope of this paper. We will just mention that values around the TeV scale do not significantly affect the mass spectrum of the theory, and that they can account for values of the left-right mass insertion, i.e.

$$(\delta^U_{LR})_{i3} = \frac{v_2}{\sqrt{2}}\,\frac{(V_L^U Y_U^* V_R^{U\dagger})_{i3}}{\langle m_{\tilde q}\rangle^2}\,, \tag{5.13}$$

as small as 0.02 and 0.08 for $(\delta^U_{LR})_{13}$ and $(\delta^U_{LR})_{23}$ respectively, which are consistent with the bounds coming from the measurement of $b \to s\gamma$ branching fraction. (Note that these δ's also affect the decay $b \to sl^+l^-$.) Indeed, according to ref. [28], $(\delta^U_{LR})_{23}$ is constrained to be $|(\delta^U_{LR})_{23}| \lesssim 3(\langle m_{\tilde q}\rangle/500\,\text{GeV})^2$, which is only useful if $\langle m_{\tilde q}\rangle \lesssim 300\,\text{GeV}$. Similarly, the chargino contribution to B^0_d–$\bar B^0_d$ mixing leads to the constraint $(\delta^U_{LR})_{13} \lesssim 0.1(\langle m_{\tilde q}\rangle/500\,\text{GeV})$ [31].[8]

[8]Note that the requirement of vacuum stability also leads to upper bounds on $(\delta^U_{LR})_{i3}$ $(i = 1, 2)$ [36]. The nominal values used in our numerical analysis are compatible with these upper limits.

At this point, a few remarks are in order regarding previous studies contained in the literature. As we have already mentioned, the diagrams depicted in figure 5 give the leading contributions to $\Im\mathcal{M}_{12}$, in agreement with refs. [11, 13]. The final form of our result, however, differs from that of previous works in several ways. First, we have used the mass insertion approximation to compute the dominant contributions to ϵ_K, which enables us to use other largely model-independent constraints on the off-diagonal elements of the squark mass matrices. Second, we have considered a non-trivial structure for the LR squark mass-squared terms, which proved to be crucial in order to have indirect CP violation within the present model. More importantly, our parameter space is severely constrained since in the calculation of the various contributions to ϵ_K we have considered only those sets of parameters that correspond to true minima of the Higgs potential.

We conclude this section with several comments on large CP phases and their implications for the EDM's of electron and neutron.

5.4 Remarks on the electric dipole moments

As seen from table 1, the CP phase of the gauge singlet field, ϕ_N, is accompanied by small values of λ so that the EDM constraints are less stringent [37]. Further, the SUSY contributions to the EDM's depend only on flavour-diagonal entries in the squark mass matrix, whereas the expression for ϵ_K involves flavour off-diagonal elements. However, it is evident from eq. (4.2) that there is an overall CP-violating phase which is independent of the new flavour structure. Apart from the requirement that $A_U^{ij}|_{i=j} \ll A_U^{ij}|_{i\neq j}$, a conceivable way to suppress the EDM's includes large cancellations between different SUSY contributions and a SUSY particle spectrum with masses of scalar quarks and gauginos in the TeV range. Given the analytic results for the contributions to the EDM's of electron and neutron mediated by photino and gluino [33], together with the sets of parameters displayed in table 1 and the present experimental results of $d_n < 6.3 \times 10^{-26}\, e\,\mathrm{cm}$ (90% C.L.) and $d_e = 1.8 \times 10^{-27}\, e\,\mathrm{cm}$ [21], the photino and gluino masses are required to satisfy $0.5\,\mathrm{TeV} \lesssim m_{\tilde{\gamma}} \lesssim 2\,\mathrm{TeV}$ and $2\,\mathrm{TeV} \lesssim m_{\tilde{g}} \lesssim 6\,\mathrm{TeV}$.[9]

Such a hierarchy in the soft gaugino masses requires further discussion. We first note that masses of the superpartners of about 1 TeV may be in conflict with the cosmological relic density. In addition, such a scale for the $\tilde{\gamma}$ and the $\tilde{g}$ masses is rather unnatural, since the masses of the squarks and W-ino are typically of the order 100–300 GeV in the model under consideration. [Similar conclusions can be drawn if we take into account the chargino and neutralino contributions (see, e.g. ref. [23]).] Finally, we stress that the above-mentioned hierarchy for the scalar partners leads to an unacceptable scenario for the lightest supersymmetric particle (LSP). In this

[9] For simplicity, we have assumed $m_{\tilde{l}} = m_{\tilde{q}}$. Note that the constraint on $m_{\tilde{g}}$ is relaxed if we allow for an order-of-magnitude variation in the value of the hadronic matrix elements.

case, the LSP would be either charged (one of the lightest squarks or a chargino), or would have a non-zero lepton number. To summarize, it appears that unless the parameters of the model are fine-tuned, there will be large contributions to the EDM's of electron and neutron, in potential conflict with experiment.[10]

6. Conclusions

The origin of CP violation, in particular, the question of whether CP is explicitly or spontaneously broken, is an important issue in particle physics. In this paper, we have studied spontaneous CP violation in the context of the NMSSM, demonstrating that it is possible to generate sufficient CP violation in order to account for the magnitude of ϵ_K. We have emphasized that the NMSSM with spontaneous CP violation is a natural framework to discuss the possibility of having CP violation that arises entirely from SUSY interactions. This is due to the fact that in the above-mentioned scenario the CKM matrix is automatically real.

By performing a complete and systematic study of the Higgs potential in the NMSSM with spontaneous CP violation, we have shown that the minimisation of the most general Higgs potential leads to an acceptable mass spectrum which is accompanied by large CP-violating phases. We have argued that the lightest neutral Higgs boson can have a reduced coupling to the Z^0 due to the additional singlet field, and thus may escape detection at LEP. We have shown that it is possible to have a mass of the lightest neutral Higgs boson of about 125 GeV. However, such values for the Higgs-boson mass require a relatively large value for $M_{\rm SUSY}$ of the order $O(1\,\mathrm{TeV})$. By contrast, a rather low SUSY scale with $M_{\rm SUSY} \approx 300\,\mathrm{GeV}$ (i.e. light squark and W-ino masses) and a non-trivial flavour structure of the soft SUSY-breaking trilinear couplings A_U^{i3} $(i = 1, 2)$ are required in order to account for the observed CP violation in K^0–$\bar{K}^0$ mixing. As a consequence, the parameter space is severely constrained and the mass of the lightest Higgs boson is further diminished, and it turns out to be no greater than $\sim 100\,\mathrm{GeV}$ for the case of low $\tan\beta$ ($\lesssim 10$).

As far as large CP phases are concerned, we have argued that it may be difficult to reconcile the large-phase solution with the severe constraints on the EDM's of electron and neutron. Although we do not exclude the possibility of cancellations between different contributions, we find this highly unlikely in view of the constrained parameter space. Therefore, the implications of the EDM bounds on the parameter space, as well as the implied LSP scenario, are a great challenge for SUSY models with spontaneous CP violation (at least within the minimal model we have considered here).

[10] As far as the B system is concerned, we merely remark that the model can accommodate a large CP asymmetry $a_{J/\psi}$, i.e. a large value of $\sin 2\beta$ (see, e.g. ref. [10]).

JHEP07(2001)027

Acknowledgments

One of us (A.M.T.) would like to thank Ricardo González Felipe for useful discussions, and the Theory Division at CERN for its kind hospitality during the final stage of this work. This project was supported in part by the TMR Network of the EC under contract ERBFMRX-CT96-0090. A.M.T. acknowledges support by 'Fundação para a Ciência e Tecnologia' under grant PRAXIS XXI BD/11030/97.

References

[1] N. Cabibbo, *Unitary symmetry and leptonic decays*, *Phys. Rev. Lett.* **10** (1963) 531;
M. Kobayashi and K. Maskawa, *CP violation in the renormalizable theory of weak interaction*, *Prog. Theor. Phys.* **49** (1973) 652.

[2] A.G. Cohen, D.B. Kaplan, and A.E. Nelson, *Progress in electroweak baryogenesis*, *Annu. Rev. Nucl. Part. Sci.* **43** (1993) 27;
M.B. Gavela, M. Lozano, J. Orloff and O. Péne, *Standard model CP-violation and baryon asymmetry. Part 1: zero temperature*, *Nucl. Phys.* **B 430** (1994) 345 [`hep-ph/9406288`]; *ibid.* **B 430** (1994) 382;
A.D. Dolgov, *Baryogenesis, 30 years after*, `hep-ph/9707419`;
V.A. Rubakov and M.E. Shaposhnikov, *Electroweak baryon number nonconservation in the early universe and in high-energy collisions*, *Usp. Fiz. Nauk* **166** (1996) 493 [*Phys. Usp.* **39** (1996) 461] [`hep-ph/9603208`].

[3] T.D. Lee, *A theory of spontaneous T violation*, *Phys. Rev.* **D 8** (1973) 1226.

[4] G.C. Branco, *Spontaneous CP nonconservation and natural flavor conservation: a minimal model*, *Phys. Rev.* **D 22** (1980) 2901; *Spontaneous CP-violation in theories with more than four quarks*, *Phys. Rev. Lett.* **44** (1980) 504.

[5] N. Maekawa, *'Spontaneous' CP-violation in minimal supersymmetric standard model*, *Phys. Lett.* **B 282** (1992) 387;
A. Pomarol, *Higgs sector CP violation in the minimal supersymmetric model*, *Phys. Lett.* **B 287** (1992) 331, [`hep-ph/9205247`];
N. Haba, *Can the Higgs sector trigger CP violation in the MSSM?*, *Phys. Lett.* **B 398** (1997) 305, [`hep-ph/9609395`];
O.C.W. Kong and F.L. Lin, *On radiative CP violation in (supersymmetric) two Higgs doublet models*, *Phys. Lett.* **B 419** (1998) 217, [`hep-ph/9707433`];
O. Lebedev, *On spontaneous CP-violation in the Higgs sector*, *Eur. Phys. J.* **C 4** (1998) 363.

[6] H. Georgi and A. Pais, *CP-violation as a quantum effect*, *Phys. Rev.* **D 10** (1974) 1246.

[7] J. Ellis, J.F. Gunion, H.E. Haber, L. Roszkowski and F. Zwirner, *Higgs bosons in a nonminimal supersymmetric model*, *Phys. Rev.* **D 39** (1989) 844;

JHEP07(2001)027

S.F. King and P.L. White, *Resolving the constrained minimal and next-to-minimal supersymmetric standard models*, *Phys. Rev.* **D 52** (1995) 4183 [hep-ph/9505326]; *Nonminimal supersymmetric Higgs bosons at LEP-2*, *Phys. Rev.* **D 53** (1996) 4049 [hep-ph/9508346].

[8] M. Drees, *Supersymmetric models with extended Higgs sector*, *Int. J. Mod. Phys.* **A 4** (1989) 3635;
U. Ellwanger, M. Rausch de Traubenberg, and C.A. Savoy, *Higgs phenomenology of the supersymmetric model with a gauge singlet*, *Z. Physik* **C 67** (1995) 665 [hep-ph/9502206];
U. Ellwanger, M. Rausch de Traubenberg and C.A. Savoy, *Phenomenology of supersymmetric models with a singlet*, *Nucl. Phys.* **B 492** (1997) 21 [hep-ph/9611251].

[9] K.S. Babu and S.M. Barr, *Spontaneous CP violation in the supersymmetric Higgs sector*, *Phys. Rev.* **D 49** (1994) 2156;
N. Haba, M. Matsuda and M. Tanimoto, *Spontaneous CP violation and Higgs masses in the next-to-minimal supersymmetric model*, *Phys. Rev.* **D 54** (1996) 6928 [hep-ph/9512421];
S.W. Ham, S.K. Oh and H.S. Song, *Spontaneous violation of the CP symmetry in the Higgs sector of the next-to-minimal supersymmetric model*, *Phys. Rev.* **D 61** (2000) 055010 [hep-ph/9910461].

[10] O. Lebedev, *Constraining SUSY models with spontaneous CP violation via $B \to \psi K_S$*, *Int. J. Mod. Phys.* **A 15** (2000) 2987 [hep-ph/9905216].

[11] O. Lebedev, *Probing spontaneous CP-violation in supersymmetric models via B decays*, *Phys. Lett.* **B 452** (1999) 294 [hep-ph/9812501].

[12] J.C. Romão, *Spontaneous CP violation in SUSY models: a no-go theorem*, *Phys. Lett.* **B 173** (1986) 309.

[13] A. Pomarol, *Spontaneous CP-violation in supersymmetric theories*, *Phys. Rev.* **D 47** (1993) 273 [hep-ph/9208205].

[14] S.A. Abel and J.M. Frère, *Could the MSSM have no CP-violation in the CKM matrix?*, *Phys. Rev.* **D 55** (1997) 1623 [hep-ph/9608251].

[15] S. Khalil, T. Kobayashi and A. Masiero, *CP-violation in supersymmetric model with nondegenerate A-terms*, *Phys. Rev.* **D 60** (1999) 075003 [hep-ph/9903544];
D.A. Demir, A. Masiero and O. Vives, *CP conserving constraints on supersymmetric CP violation in the MSSM*, *Phys. Rev.* **D 61** (2000) 075009, [hep-ph/9909325].

[16] S. Baek, J.H. Jang, P. Ko and J.H. Park, *Fully supersymmetric CP-violations in the kaon system*, *Phys. Rev.* **D 62** (2000) 117701 [hep-ph/9907572];
M. Brhlik, L.L. Everett, G.L. Kane, S.F. King and O. Lebedev, *Can supersymmetry soft phases be the source of all CP-violation?*, *Phys. Rev. Lett.* **84** (2000) 3041 [hep-ph/9909480].

[17] G.C. Branco, F. Cagarrinho and F. Krüger, *A real CKM matrix and physics beyond the standard model*, *Phys. Lett.* **B 459** (1999) 224 [hep-ph/9904379].

[18] A.T. Davies, C.D. Froggatt and R.G. Moorhouse, *Electroweak baryogenesis in the next to minimal supersymmetric model*, *Phys. Lett.* **B 372** (1996) 88 [hep-ph/9603388];
A.T. Davies, C.D. Froggatt and A. Usai, *Proceedings of the International Europhysics Conference on High-Energy Physics*, edited by D. Lellouch, Springer-Verlag, Berlin 1999, [hep-ph/9712501]; *CP-violation in the next to minimal supersymmetric standard model*, hep-ph/9902476.

[19] S.J. Huber and M.G. Schmidt, *Electroweak baryogenesis: concrete in a SUSY model with a gauge singlet*, hep-ph/0003122.

[20] Y.B. Zeldovich, I.Y. Kobzarev and L.B. Okun, *Cosmological consequences of the spontaneous breakdown of discrete symmetry*, *Zh. Eksp. Teor. Fiz.* **67** (1974) 3;
S.A. Abel, S. Sarkar and P.L. White, *On the cosmological domain wall problem for the minimally extended supersymmetric standard model*, *Nucl. Phys.* **B 454** (1995) 663 [hep-ph/9506359];
S.A. Abel, *Destabilizing divergences in the NMSSM*, *Nucl. Phys.* **B 480** (1996) 55 [hep-ph/9609323];
C. Panagiotakopoulos and K. Tamvakis, *New minimal extension of MSSM*, *Phys. Lett.* **B 469** (1999) 145 [hep-ph/9908351].

[21] Particle Data Group collaboration, D.E. Groom et al., *Review of particle physics*, *Eur. Phys. J.* **C 15** (2000) 1.

[22] T. Ibrahim and P. Nath, *The neutron and the lepton EDMs in MSSM, large CP-violating phases and the cancellation mechanism*, *Phys. Rev.* **D 58** (1998) 111301 [hep-ph/9807501]; *ibid.* **60** (1999) 099902 (E);
M. Brhlik, G.J. Good and G.L. Kane, *Electric dipole moments do not require the CP violating phases of supersymmetry to be small*, *Phys. Rev.* **D 59** (1999) 115004 [hep-ph/9810457];
A. Bartl, T. Gajdosik, W. Porod, P. Stockinger and H. Stremnitzer, *Electron and neutron electric dipole moments in the constrained MSSM*, *Phys. Rev.* **D 60** (1999) 073003 [hep-ph/9903402].

[23] S. Pokorski, J. Rosiek and C.A. Savoy, *Constraints on phases of supersymmetric flavour conserving couplings*, *Nucl. Phys.* **B 570** (2000) 81 [hep-ph/9906206].

[24] See, for example, S. Khalil, T. Kobayashi and O. Vives, *EDM-free supersymmetric CP-violation with non-universal soft terms*, *Nucl. Phys.* **B 580** (2000) 275 [hep-ph/0003086].

[25] M. Bastero-Gil, C. Hugonie, S.F. King, D.P. Roy and S. Vempati, *Does LEP prefer the NMSSM?*, *Phys. Lett.* **B 489** (2000) 359 [hep-ph/0006198];
G.L. Kane, S.F. King and L.-T. Wang, *What will we learn if a Higgs boson is found?*, hep-ph/0010312, and references therein.

[26] U. Ellwanger, M. Rausch de Traubenberg and C.A. Savoy, *Particle spectrum in supersymmetric models with a gauge singlet*, *Phys. Lett.* **B 315** (1993) 331 [hep-ph/9307322];
J.F. Gunion, H.E. Haber and T. Moroi, *Will at least one of the Higgs bosons of the next-to-minimal supersymmetric extension of the standard model be observable at LEP2 or the LHC?*, hep-ph/9610337;
B. Grzadkowski, J.F. Gunion and J. Kalinowski, *Finding the CP-violating Higgs bosons at e^+e^- colliders*, *Phys. Rev.* **D 60** (1999) 075011 [hep-ph/9902308];
U. Ellwanger and C. Hugonie, *Masses and couplings of the lightest Higgs bosons in the $(m+1)SSM$*, hep-ph/9909260.

[27] J. Bernabéu, G.C. Branco and M. Gronau, *CP restrictions on quark mass matrices*, *Phys. Lett.* **B 169** (1986) 243.

[28] M. Misiak, S. Pokorski and J. Rosiek, *Supersymmetry and fcnc effects*, in *Heavy Flavours II*, A.J. Buras and M. Lindner eds., World Scientific, Singapore 1998, hep-ph/9703442.

[29] J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, *Errata for the Higgs Hunter's guide*, hep-ph/9302272;
J.F. Gunion and H.E. Haber, *Higgs bosons in supersymmetric models, 1*, *Nucl. Phys.* **B 272** (1986) 1, erratum *ibid.* **B 402** (1993) 567;
H.E. Haber and G.L. Kane, *The search for supersymmetry: probing physics beyond the standard model*, *Phys. Rep.* **117** (1985) 75.

[30] L.J. Hall, V.A. Kostelecky and S. Raby, *New flavor violations in supergravity models*, *Nucl. Phys.* **B 267** (1986) 415;
see also M. Dugan, B. Grinstein and L. Hall, *CP-violation in the minimal $N = 1$ supergravity theory*, *Nucl. Phys.* **B 255** (1985) 413.

[31] A.J. Buras, A. Romanino and L. Silvestrini, *$K \to \pi\nu\bar{\nu}$: a model independent analysis and supersymmetry*, *Nucl. Phys.* **B 520** (1998) 3 [hep-ph/9712398].

[32] See, for example, G. Buchalla, A.J. Buras and M.E. Lautenbacher, *Weak decays beyond leading logarithms*, *Rev. Mod. Phys.* **68** (1996) 1125 [hep-ph/9512380].

[33] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, *A complete analysis of FCNC and CP constraints in general SUSY extensions of the standard model*, *Nucl. Phys.* **B 477** (1996) 321 [hep-ph/9604387].

[34] M. Ciuchini et al., *ΔM_K and ϵ_K in SUSY at the next-to-leading order*, *J. High Energy Phys.* **10** (1998) 008 [hep-ph/9808328];
A.J. Buras, M. Misiak and J. Urban, *Two-loop QCD anomalous dimensions of flavour-changing four-quark operators within and beyond the standard model*, *Nucl. Phys.* **B 586** (2000) 397 [hep-ph/0005183].

[35] D.A. Demir, *Dynamical relaxation of the CP phases in next-to-minimal supersymmetry*, *Phys. Rev.* **D 62** (2000) 075003 [hep-ph/9911435].

JHEP07(2001)027

[36] J.A. Casas and S. Dimopoulos, *Stability bounds on flavor-violating trilinear soft terms in the MSSM*, *Phys. Lett.* **B 387** (1996) 107 [hep-ph/9606237].

[37] M. Matsuda and M. Tanimoto, *Explicit CP-violation of the Higgs sector in the next-to-minimal supersymmetric standard model*, *Phys. Rev.* **D 52** (1995) 3100 [hep-ph/9504260].